CALCULATE THE TOP QUARK MASS
TEACHER NOTES

DESCRIPTION

In 1994 physicists at Fermilab announced the discovery of a new particle dubbed “Top Quark.” The discovery of this particle completed the Standard Model chart until discovery of the Higgs Boson was announced in 2012. Despite the use of gigantic detectors using ground-breaking technology, the data analysis involved concepts that are a part of the standard curriculum for high school physics.

In this activity, students use momentum conservation, energy conservation and two-dimensional vector addition to calculate the mass of the heaviest of the six known quarks. They gather data from data plots from the DØ experiment at Fermilab. The events were chosen carefully; all of the decay products moved in a plane perpendicular to the beam. This allows for analysis to take place in two dimensions instead of the events that are best displayed in three dimensions. The use of two dimensional vector analysis fits in nicely during either a conservation laws unit or as a practice problem for the vector analysis unit.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

4. Analyzing and interpreting data
5. Using mathematics and analytical thinking
6. Constructing explanations
7. Engaging in arguments from evidence

Crosscutting Concepts

1. Observed patterns
4. Systems and system models
5. Energy and matter
7. Stability and change

Common Core Literacy Standards

Reading

9-12.3 Follow precisely a complex multistep procedure . . .
9-12.4 Determine the meaning of symbols, key terms . . .
9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP2. Reason abstractly and quantitatively
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

ENDURING UNDERSTANDINGS

Particle physicists use conservation of energy and momentum to discovery the mass of fundamental particles.

LEARNING OBJECTIVES

Students will know and be able to:

- Use conservation of momentum and energy to determine the mass of a top quark.

- Explain the importance of identifying the missing momentum carried away from the event by the neutrino.
- Describe the properties of a neutrino that make it impossible to detect in the DØ detector.
- Explain the importance of considering the results of several experiments before announcing discoveries.

PRIOR KNOWLEDGE

Students should be able to:

- Add vectors in two dimensions.
- Use energy and momentum units common to particle physics: Momentum–eV/c, Energy–eV/c^2.

BACKGROUND MATERIAL

Useful links to describe how detectors work:

- https://home.cern/about/how-detector-works
- http://lutece.fnal.gov/Papers/PhysNews95.html

RESOURCES/MATERIALS

Students will need a ruler, a protractor, calculator or spreadsheet and data from the following link:
https://drive.google.com/file/d/1oGOnpDnX29EaS9gLl3mhiO99sTo2E61Z/view

IMPLEMENTATION

This activity was developed by physics teacher Bob Grimm. Students use printed event images, ruler and protractor to analyze the data. This activity requires averaging as many as ten data points per class to yield a more accurate result.

The students analyze DØ top-quark event images, one real and three Monte Carlo, chosen because the events happened in the plane perpendicular to the beam line. This means that students will get good results using two-dimensional vector analysis. Students use a ruler and protractor to add particle momentum vectors, discover the magnitude of the missing neutrino momentum and calculate the mass of the top quark.

When Bob used this activity, his students were studying conservation laws, so the activity fit right in. They didn't know about quarks, etc., but they spent some time, before the lab day, looking at images taken from the website that describe the top quark experiment. So, with their particle questions addressed, they were able to focus on the vector addition portion of the activity. This approach is a good use of instructional time; on lab day, the students' questions about vectors and momentum were not confused by questions about particles. This is not to say they did not have questions about particles. Indeed, the questions they had were fairly sophisticated. But they did not seem distracted by details of the experiment.

Bob recommends keeping in mind the following "I" ideas while leading your students through this activity:

- Invest time in describing the experiment.
- Ignore errors in the direction of the neutrino momentum vector.
- Integrate your students' results. Averaging a large set of data is critical.
- Indistinguishable units. Near the speed of light, mass = momentum = energy.

The key to finding the momentum carried away by the neutrino is to determine the "missing transverse momentum (Pt)." Since the detector can't detect neutrinos—they barely interact with matter—have your students look at all of the momentum recorded in the event and then apply momentum conservation to determine what is needed to make the system's net momentum zero. Recall that energy and momentum are equal at these energies.

If you have never done this before, the process is:

- Use a protractor to find the angle θ that the lines through the centers of all jets and the muon tracks make with the x-axis.
- The magnitude of the momentum p for all the jets and muons is given on the plot. Find $p_x = p\cos(\theta)$ and $p_y = p\sin(\theta)$ for all jets and muons.
- Find $p_{x\ total\_observed}$ and $p_{y\ total\_observed}$.
- Recall that the center of mass momentum before the collision is zero, there since momentum is conserved, the vector sum of momenta after the collision must also be zero. Since we can account for all of the momentum except the missing neutrino momentum, the x and y component of the neutrino momentum are

  $p_{neutrino\ x} = -p_{x\ total\_observed}$ and $p_{neutrino\ y} = -p_{y\ total\_observed}$.
- The magnitude of the neutrino momentum is $p_{neutrino} = (p_{neutrino\ x}^2 + p_{neutrino\ y}^2)^{1/2}$.
- Add up the energies of all jets, muons, and the neutrino to find the rest energy of the top/anti-top pair.
- Since the event is a top/anti-top event, total energy divided by two results in the mass of the top quark in units of GeV/c^2.

ASSESSMENT

The following questions and sample scoring rubric may be used for assessment:

- Explain the mathematical model for finding the missing momentum carried off by the neutrino.
  - *Choosing a coordinate system*
  - *Measuring the angle of all vectors relative to the chosen x-axis*
  - *Correctly determining the x-component and y-component of each momentum vector*
  - *Finding the sum of the x-components and y-components*
  - *Indicating that the vector components should add to zero, determining the x-component momentum and y-component momentum of the neutrino needed to make the components sums equal to zero*
- Using the neutrino x-component and y-component to determine the magnitude of the missing neutrino momentum.
- Determine the energy of the neutrino must be the same as the magnitude of the momentum of the neutrino when appropriate units are chosen.
  - *Start with Einstein's equation* $E^2 = p^2c^2+(mc^2)^2$
  - *In the correct units, the reduces to* $E^2 = p^2+m^2$
  - *The mass of the neutrino is negligible at these energy levels so* $E=p$
- Explain how conservation of energy is used to determine the mass of the top quark.
  - *the sum of the energy of the jets, muons, and neutrino must equal the mass of the top quark/top anti-quark pair.*

- Describe the properties of a neutrino that make it impossible to detect in the DØ detector.
    - *The neutrino has no charge and therefore does not interact with the tracking section of the detector or the electromagnetic calorimeter*
    - *The neutrino has such small mass, it does not interact with matter and therefore will not be detectable in the hadron calorimeter or the muon detector sections of the detector*
- Compare your individual result with the value determined from the class average.
    - *Were their outliers?*
    - *How close was the class average to the value determined by scientists?*
    - *Was the value determined by scientists within the range of the values found by your class?*
- Use these results to describe why scientists want repeatability of results before announcing discoveries.